Section 1.2

Properties of Signals

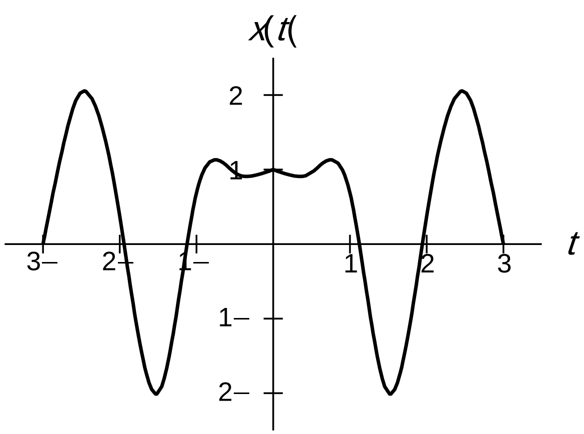
- A function x is said to be even if it satisfies

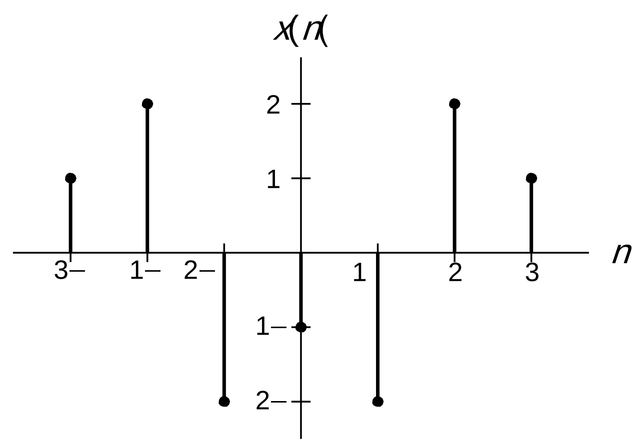
$$x(t) = x(-t) \quad \text{for all } t.$$

- A sequence x is said to be even if it satisfies

$$x(n) = x(-n) \quad \text{for all } n.$$

- Geometrically, the graph of an even signal is *symmetric* about the origin.
- Some examples of even signals are shown below.





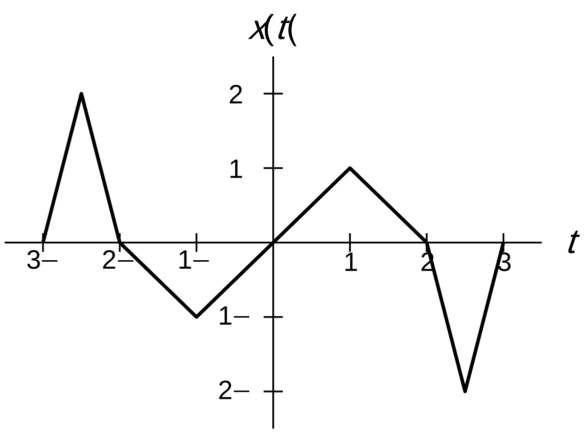
- A function x is said to be odd if it satisfies

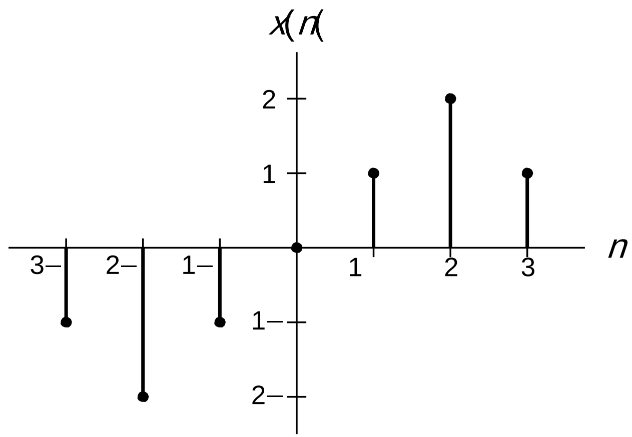
$$x(t) = -x(-t) \quad \text{for all } t.$$

- A sequence x is said to be odd if it satisfies

$$x(n) = -x(-n) \quad \text{for all } n.$$

- Geometrically, the graph of an odd signal is *antisymmetric* about the origin.
- An odd signal x must be such that $x(0) = 0$. Some
- examples of odd signals are shown below.





- A function x is said to be periodic with period T (or T-periodic) if, for some strictly-positive real constant T, the following condition holds:

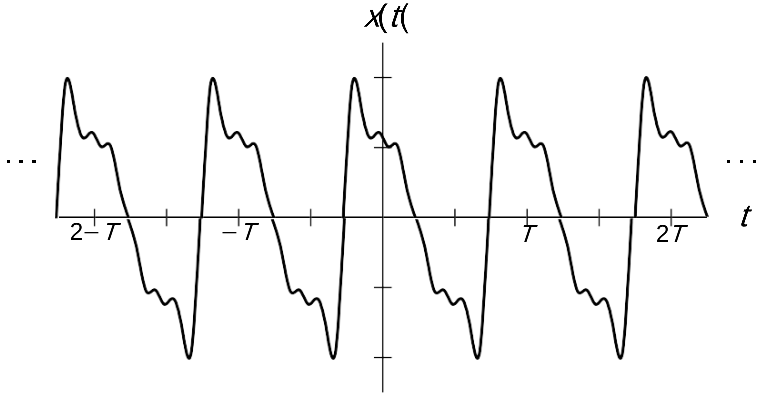
$$x(t) = x(t + T) \quad \text{for all } t.$$

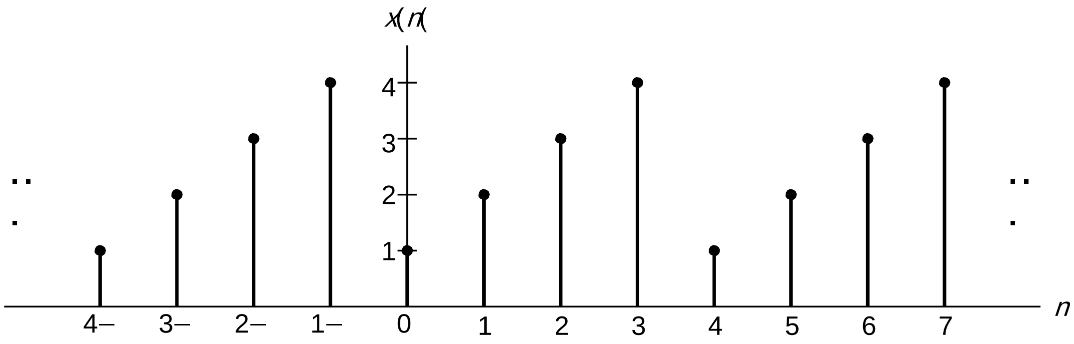
- A T-periodic function x is said to have frequency $\frac{1}{T}$ and angular frequency $\frac{2\pi}{T}$.
- A sequence x is said to be periodic with period N (or N-periodic) if, for some strictly-positive integer constant N, the following condition holds:

$$x(n) = x(n + N) \quad \text{for all } n.$$

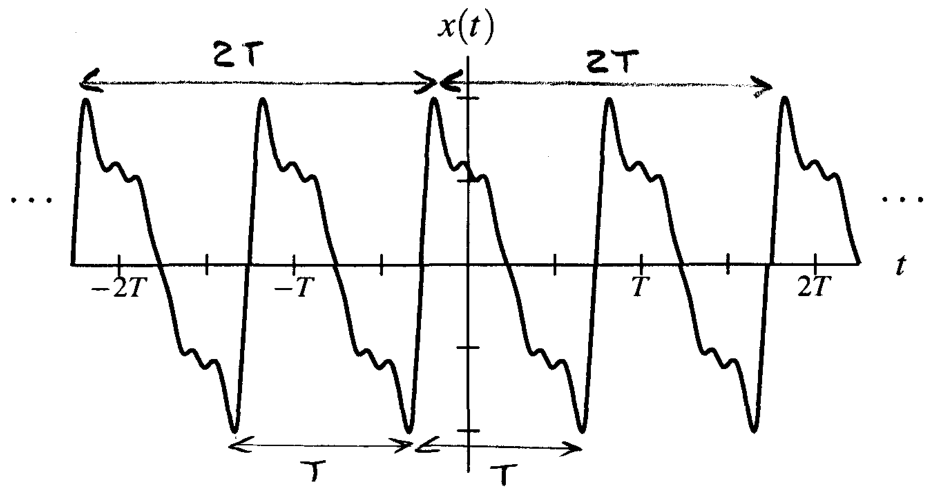
- An N-periodic sequence x is said to have frequency $\frac{1}{N}$ and angular frequency $\frac{2\pi}{N}$.
- A function/sequence that is not periodic is said to be aperiodic.

- Some examples of periodic signals are shown below.





- The period of a periodic signal is *not unique*. That is, a signal that is periodic with period T is also periodic with period kT, for every (strictly) positive integer k.



- The smallest period with which a signal is periodic is called the fundamental period and its corresponding frequency is called the fundamental frequency.

Part 2

Continuous-Time (CT) Signals and Systems

Section 2.1

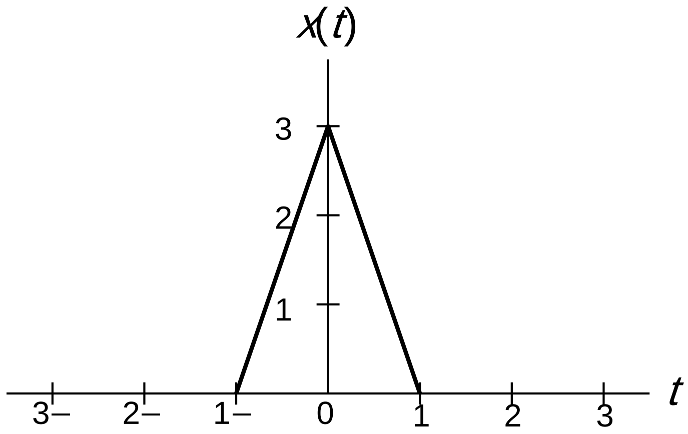
Independent- and Dependent-Variable Transformations

- Time shifting (also called translation) maps the input signal x to the output signal y as given by

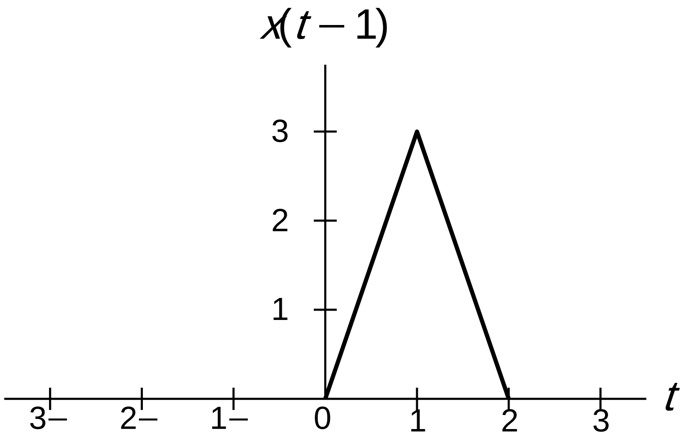
$$y(t) = x(t-b),$$

where b is a real number.

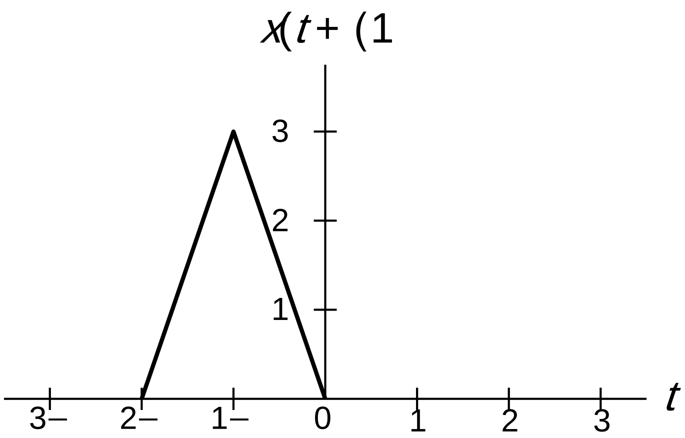
- Such a transformation shifts the signal (to the left or right) along the time axis.
- If $b > 0$, y is *shifted to the right* by $|b|$, relative to x (i.e., delayed in time). If
- $b < 0$, y is *shifted to the left* by $|b|$, relative to x (i.e., advanced in time).



x(t)
3
2
1
t
3–
2–
1–
0
1
2
3



x(t – 1)
3
2
1
t
3–
2–
1–
0
1
2
3

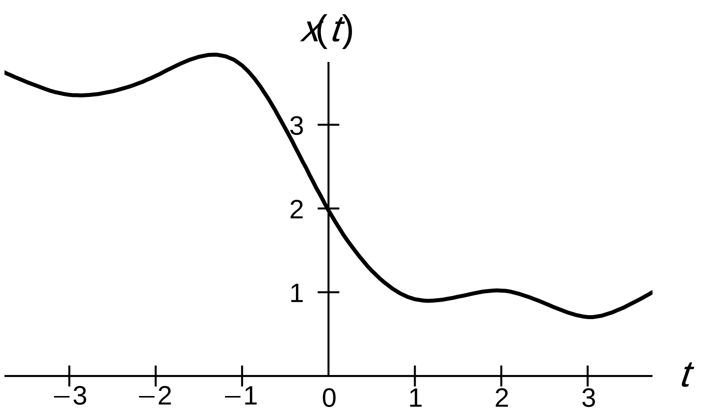


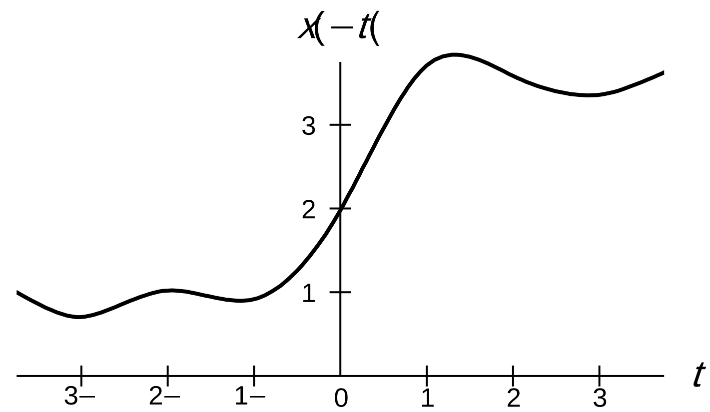
x(t + (1
3
2
1
t
3–
2–
1–
0
1
2
3

- Time reversal (also known as reflection) maps the input signal x to the output signal y as given by

$$y(t) = x(-t).$$

- Geometrically, the output signal y is a reflection of the input signal x about the (vertical) line $t = 0$.

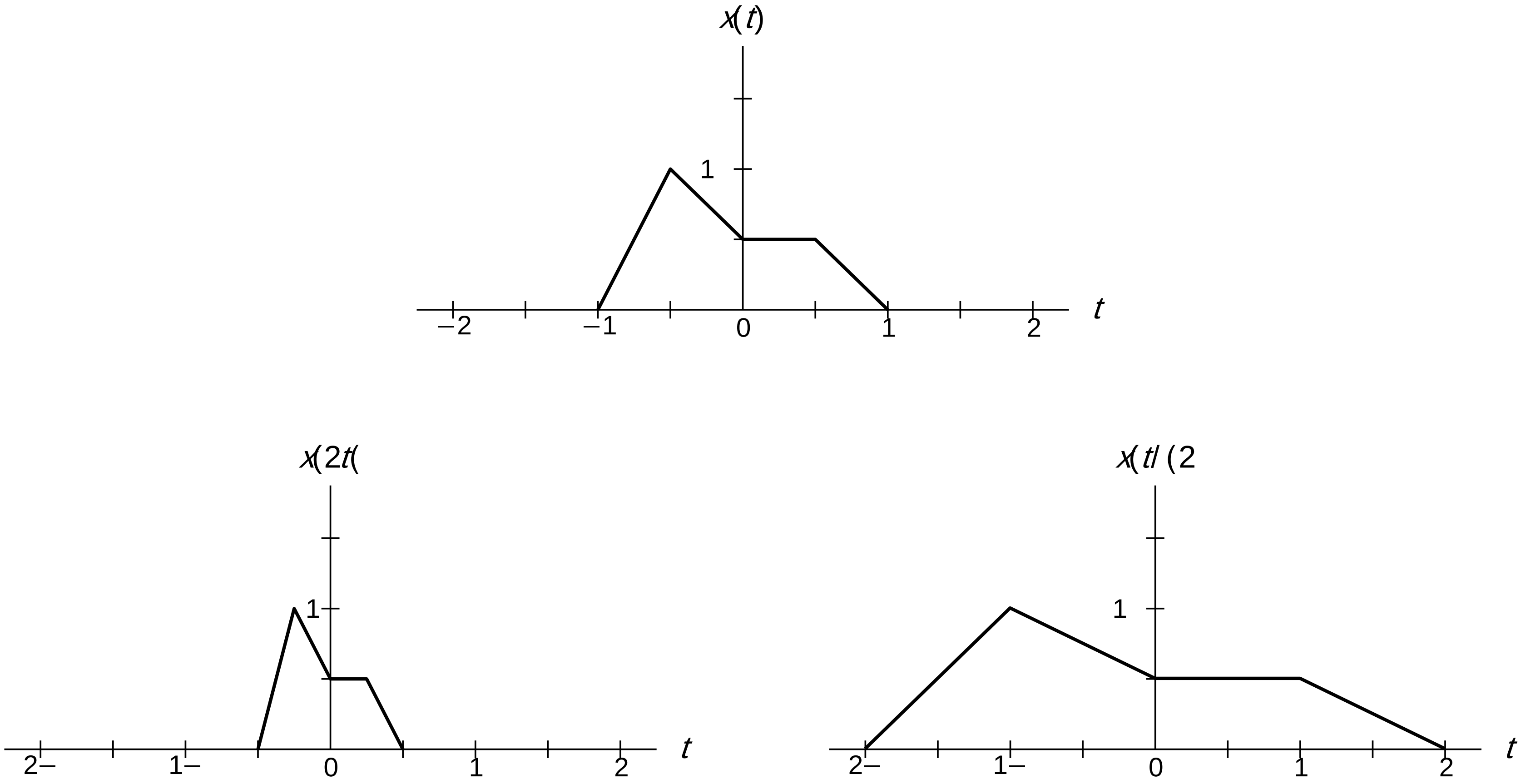




- Time compression/expansion (also called dilation) maps the input signal x to the output signal y as given by

$$y(t) = x(at),$$

where a is a *strictly positive* real number.
- Such a transformation is associated with a compression/expansion along the time axis.
- If $a > 1$, y is *compressed* along the horizontal axis by a factor of a, relative to x.
- If $a < 1$, y is *expanded* (i.e., stretched) along the horizontal axis by a factor of $\frac{1}{a}$, relative to x.



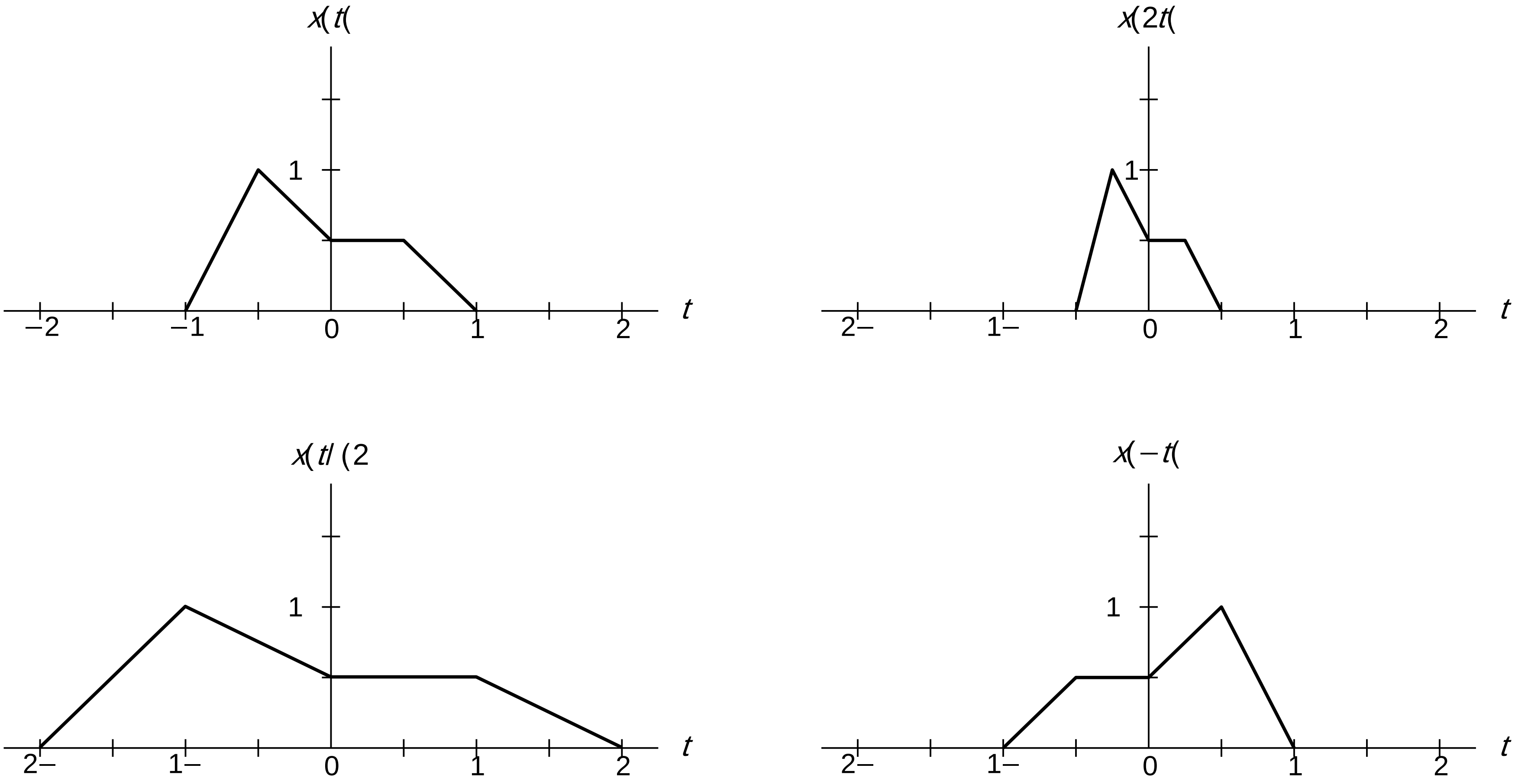
x(t)
1
−2
−1
0
1
2
t
x(2t(
1
2−
1−
0
1
2
t
x(t/ (2
1
2−
1−
0
1
2
t

- Time scaling maps the input signal x to the output signal y as given by

$$y(t) = x(at),$$

where a is a *nonzero* real number.

- Such a transformation is associated with a dilation (i.e., compression/expansion along the time axis) and/or time reversal.
- If $|a| > 1$, the signal is *compressed* along the time axis by a factor of $|a|$.
- If $|a| < 1$, the signal is *expanded* (i.e., stretched) along the time axis by a factor of $\frac{1}{|a|}$.
- If $|a| = 1$, the signal is neither expanded nor compressed. If
- $a < 0$, the signal is also time reversed.
- Dilation (i.e., expansion/compression) and time reversal *commute*.
- Time reversal is a special case of time scaling with $a = -1$; and time compression/expansion is a special case of time scaling with $a > 0$.


x(t(
x(2t(
x(t/ (2
x(− t(
1
t
−2
−1
0
1
2